DOCUMENT RESUME

ED 222 522 TM 820 668

AUTHOR Willson, Victor L.
TITLE Misuses of Regression Approaches to ANOVA and ANCOVA.
PUB DATE Feb 82
NOTE 14p.; Paper presented at the Annual Meeting of the Southwest Educational Research Association (Austin, TX, February 11, 1982).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Analysis of Covariance; *Analysis of Variance; Error of Measurement; Generalization; Mathematical Models; *Multiple Regression Analysis; *Research Problems; Research Utilization; Statistical Analysis; Statistical Significance

ABSTRACT

The current state of usage of regression models in analysis of variance (ANOVA) designs is empirically examined, and examples of several statistical errors made in usage are presented. The assumptions of the general linear model are that all predictors are known without error of measurement and are fixed with no replication or sample variation; in the population, errors are normally distributed independently with variance, and errors are independent of all predictors. The rules for construction of the ANOVA allow the expected mean squares to hold just as if the levels of each factor had been randomly sampled. Analysis of Covariance (ANCOVA) combines the elements of regression analysis with design, albeit in a restricted manner. The homogeneity of regression coefficients is a parameter restriction from the design view point. The regression weight associated with a given covariate level is discussed. Most regression approaches to ANOVA and ANCOVA assume a fixed factor model under all design specifications. A major oversight of expected mean squares has contributed to the current lack of concern for the level of generalizability warranted from the design specification. Aptitude treatment interaction models are examined. (Author/CM)

Misuses of Regression Approaches to ANOVA and ANCOVA

Victor L. Willson

Texas A&M University

Paper presented at the Southwest Educational Research Association Annual Meeting, Austin, Texas, February 11, 1982.

Misuses of Regression Approaches to ANOVA and ANCOVA

The instruction of several generations of education graduate students in research design statistics was based on Lindquist (1953) and his logical succession: Winer (1962, 1971), Kirk (1968), Glass and Stanley (1970) and others. All stressed analysis of variance (ANOVA) using Fisherian partition of sums of squares. There was very little emphasis on regression approaches until the appearance of Ward and Jennings' (1973) and Kerlinger and Pedhazur's (1973) texts, which use regression models exclusively. These tests have apparently promoted increased use of regression models in ANOVA situations in the last several years. Willson (1980) reviewed ten years' research in the American Educational Research Journal, from 1969 to 1978, and found little use of regression approaches. Since 1978, however the technique has been extensively used, as will be reported here. It is the purpose of this paper to examine empirically the current state of usage of regression models in ANOVA designs and to list by example several statistical errors made in this usage.

Regression Assumptions. It is worth reviewing assumptions of the general linear model, for these assumptions will be referenced in light of current practice. Darlington (1968) has summarized the assumption as follows:

1. All predictors Xi are known without error of measurement;
2. All predictors Xi are fixed with no replication or sample variation;

3. In the population errors are normally distributed independently with variance σe^2;

4. Errors are independent of all Xi.

Design Assumptions. Cornefield and Tukey (1956) and Millman and Glass (1967) presented the rules for construction of the ANOVA table for the design in which a levels of factor A are drawn with equal probability from N_A possible levels, b levels of Factor B from N_B possible levels, c from N_C and so forth for each factor in the design. Random factors are defined for any $q < N_q$, and fixed factors defined for $r = N_r$. In each cell abc . . . q of the design n elements are drawn at random from a possible $N_{abc...q}$ in the population of elements. This is the urn sampling model of Cornfield and Tukey (1956; p. 917). The expected mean squares hold for this model just as if the levels of each factor had been randomly sampled.

ANCOVA Assumptions. Analysis of covariance combines the elements of regression analysis with design, albeit in a restrictive manner. From a regression point of view the covariate must be known without error, as must the treatment level (these are coded with 1, 0, or -1 in so-called dummy coding). The so-called assumption of homogeneity of regression coefficients is really just a parameter restriction from the design point of view. There could be a different covariate effect at each level of a covariate. The covariate can be considered either fixed (drug dosage maintenance levels in 100 mg increments from 0 to 1000) or random (drug levels in 10 mg increments from 0 to 1000, randomly sampled in stratified 100 mg groups). The regression weight associated with a given covariate level could be different for each level; more commonly it is assumed to be identical for all covariate levels, hence the homogeneity of coefficients, which is merely a restrictive form of a general linear model (Ward & Jennings, 1973).

The addition of a so-called covariate treatment interaction term can be thought of in design terms as addition of an interaction with (T-1) X(C-1) degrees of freedom where T is the number of treatments and C the number of covariate levels. This term is reduced in most analyses to T-1 degrees of freedom, one parameter per group. Each group has a different regression slope. This model is most commonly encountered in educational research in the aptitude-treatment interaction models of Cronbach and his co-workers (Cronbach & Snow, 1977).

Expected Mean Squares for ANCOVA. The approach to ANCOVA taken in most texts (Winer, 1971; Kirk, 1968) is to treat covariates as random factors whose variances are removed from sums of squares for the usual ANOVA. In effect the expected mean squares for ANOVA are conditional on the covariates and the expectations are so written (Winer, 1971; p. 770). What is quite clear is that the residual mean square after fit of the full model is not the appropriate error term for the covariate or covariates under the usual model. Those who use regression theory have rather casually used the difference between mean squares with and without covariate divided by mean square residual as the test of covariates' significance. Table 1 shows the mean square expectation table for a single covariate-single factor design under usual assumptions, most important of which is that treatment effects are conditional on the covariate adjustment.

Insert Table 1 About Here

Unreliability of Regression Variables. It was noted earlier that regression analysis models assume the predictors to be known without error. When error of measurement is present the assumption is violated. Glass, Peckham and Sanders (1972) have reviewed research on this violation for ANCOVA models. Rogosa (1977) has examined the effects of unreliability on interaction terms in ANCOVA models for confidence interval estimation using the Johnson-Neyman technique.

Glass et al (1972) concluded that unreliability does affect results of ANCOVA in unpredictable ways. They recommend procedures to adjust the F-statistic.

Rogosa (1977) reviewed the literatures on unreliable predictors (covariates) for within-group regression (non-homogeneous covariate slopes). Power is reduced and the Type I error rate may be changed. The situation is even more complex for two predictors (covariates), and distortion of error rates can be even more severe.

Stochastic Regression/Covariate Variables. Glass et al (1972) indicate there is no serious difficulty when a covariate or predictor is not fixed but random. Rogosa (1977) found no effect for ANCOVA models with within-group, nonhomogeneous regression slopes.

Method

All data based studies from three educational research journals were surveyed for years 1979 to 1981. The journals were American Educational Research Journal, Journal of Educational Psychology, and Educational Evaluation and Policy Analysis. Studies using ANOVA-type designs with regression-type statistical analyses were examined. Expected mean square tables were constructed where possible for each design using the procedures of Millman and Glass (1967). A comparison was made with empirical results reported for each table. Assumptions of the ANOVA and ANCOVA model used were compared with actual practice and discrepancies noted. Of special interest were model misspecification, nonhomogeneity of regression slopes for covariates, and unreliable covariates. Model misspecification is defined as incorrect specification of a factor or predictor as fixed (or random) when common practice or the author's later generalization clearly point to the opposite specification. Nonhomogeneity of regression slopes refers to the possibility of covariate-treatment interaction which was never tested. Unreliability of covariates refers to the presence of unreliable covariates, typically intelligence, achievement, or socioeconomic measures. Differential reliability may exist across treatment groups, which was not examined.

Results

From all articles published between 1979 and 1981 there were 29 in AERJ and JEP (none in EEPA) that used a regression approach to ANOVA or ANCOVA. Six of these were straightforward fixed factor ANOVAs that met all usual ANOVA assumptions. Of the remaining 23 eleven treated factors as fixed that are either usually treated as random or treated a factor as fixed and then generalized about the population from which it was drawn. Factors thus treated included teachers, classrooms, students, and school buildings.

Eight of the studies made no tests of homogeneity of regression slopes in ANCOVA models. Since many of the remaining studies were aptitute-treatment studies in which this interaction test was the major thrust of the study, the failure to test was significant in the remainder. It should be noted that few studies using straight ANCOVA tested for homogeneity either.

Insert Table 2 About Here

Of the twenty-three studies eighteen had covariates known to be unreliable. Fourteen made no attempt to test for unequal reliabilities across groups, which four made extensive use of generalizability theory to explore facet generalizability.

In all cases where mixed models should have been used (but one) the residual was used as the error term for all F-tests. In some cases it was possible to reconstruct expected mean square tables under the appropriate mixed model. It was apparent that numerous F statistics would change from significance to nonsignificance or vice-versa, changing interpretations in some instances. Even this first step reanalysis did not pursue hierarchical pooling procedures on construct quasi-F's under all models. It is possible to say that some studies need reinterpretation.

Discussion

Most regression approaches to ANOVA and ANCOVA published in two major education journals in the last three years assume a fixed factor model under all design specifications. This procedure seems unnecessarily thoughtless, although the ease of computation using a computer package such as SAS with its PROC GLM may have contributed to it. When one reviews the most commonly used regression-approach design texts (Kerlinger & Pedhazur, 1973; Ward & Jennings, 1973; Cohen & Cohen, 1975) there is no mention of expected mean squares in them. This major oversight clearly has contributed to the current lack of concern for the level of generalizability warranted from the design specification of a study. Cohen and Cohen (1975) do attempt to place all designs as fixed, however.

It is interesting that so many of the studies involved aptitude-treatment interaction (ATI). ATI models always involve covariates, the aptitudes, and factors, the treatments. The interactions form the major tests of interest, and yet in not one study with the exception of Martin, Veldman, and Anderson (1980) was a mixed model specifically examined. Given the random factors that accompany these studies the ATI's expected mean-squares surely involve more complex interactions of the random factors with aptitude covariate. This aspect needs immediate address, given the high interest in ATI research today.

Table 1

Expected Mean Square Table for ANCOVA,

One Covariate, One Factor (Fixed or Random)

	Variance						
	Model I (Covariate)					Model II (No Covariate)	
Source of Variation	σ^2_e	$\sigma^2_{\varepsilon\mid x}$	σ^2_x	^2_A	$\sigma^2_{A/x}$	$\sigma^2_{\varepsilon,x}$	σ^2_A
A		1			n	1	n
X	1		B^2				
e	1						
$\varepsilon\mid x$		1					
ε,x						1	

Model $y_{ij} = \mu + \beta(x_i - \mu_x) + \alpha_i + e_{ij}$ $\qquad y_{ij} = \mu + \alpha_i + (\varepsilon, X)_{ij}$

$$y_{ij} - \beta(x_i - \mu_x) = \mu + \alpha_i + (\varepsilon\mid X)_{ij}$$

$$\sigma^2_e = \sigma^2_\varepsilon(1 - \rho^2_w)\, fe / (fe - 1) \qquad \rho^2_w = [\underset{A}{\mathrm{CoRR}}\,(y, X)]^2$$

fe = degrees of freedom for error

Table 2

Journal Articles Using Regression Models for ANOVA

and ANCOVA with Design Specification Errors and Misuse

Article	Journal (1=AERJ 2=JEP)	Model Misspecification	Slope test	Reliability
Corno (1979)	1	X		X
Melican & Feldt (1980)	1	X		+++
Evertson et al (1980)	1			X
Alderman & Powers (1980)	1	X	X	X
Greene (1980)	1	X	X	X
Peterson et al (1980)	1	X		X
Martin, Veldman & Anderson (1980)	1	+	X	
Corno et al (1981)	1	X		X
Janicki & Peterson (1981)	1	X		+++
Beady & Hansell (1981)	1		X	X
Everston et al (1981)	1		+++	X
Pascarella et al (1981)	1		X	X
Peterson et al (1981)	1	X	X	+
Sharp (1981)	1		X	X

Article	Journal	Model Misspecification	Slope test	Reliability
Slavin (1979)	2	+	X	X
Peterson (1979)	2	X		X
Clark et al (1979)	2	X	+++	X
Corno (1980)	2	X		+++
Slavin+ (1980)	2	X		
Schunk (1981)	2		X	
White et al (1981)	2	+		X
Ross & Rakow (1981)	2	+		
Stinard & Dolphin (1981)	2	+		

Legend:

X = problem

+++ = addressed or tested

blank = not a threat or not relevant

+ unclear form of analysis

References

Cohen, J. & Cohen, P. Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1975.

Cornefield, J., & Tukey, J. W. Average values of mean squares in factorials. Ann. Math. Stat., 1956, 27, 907-949.

Cronbach, L. J., & Snow, R. E. Aptitudes and instructional methods: A handbook for research on interactions. New York: Irvington/Naiburg, 1977.

Darlington, R. B. Multiple regression in psychological research and practice. Psychological Bulletin, 1968, 69, 161-182.

Glass, G.V., Peckham, P.D., & Sanders, J.R. Consequences of failure to meet assumptions underlying the analysis of variance and covariance. Review of Educational Research, 1972, 42, 237-288.

Glass, G. V., & Stanley, J.C. Statistical Methods in Education and Psychology. Englewood Cliffs, NJ: Prentice-Hall, 1970.

Kerlinger, F. N., & Pedhazur, E. J. Multiple Regression in Behavioral Research. New York: Holt, Rinehart, and Winston, 1973.

Kirk, R. E. Experimental design: procedures for the behavioral sciences. Belmont, CA: Brooks-Cole Publ., Co., 1968.

Millman, J., & Glass, G. V. Roles of thumb for writing the ANOVA table. J. Educ. Meas., 1967, 4, 41-51.

Rogosa, D. R. Some results for the Johnson-Heyman technique (doctoral dissertation, Stanford University, 1977). (Xerox University Microfilms No. 78-2225).

Ward, J. H., & Jennings, E. Introduction to Linear Models. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1973.

Willson, V. L. Research techniques in AERJ articles: 1969-1978. Educational Researcher, 1980, 9, (6), 5-10.

Winer, B. J. Statistical Principles in Experimental Design. New York: McGraw-Hill, 1971.